The Farey series devil’s staircase: Connection to dynamical systems, statistical physics, music theory and music perception?

RICHARD KRANTZ, Metropolitan State College of Denver, JACK DOUTHETT, University of New Mexico, JULYAN CARTWRIGHT, Instituto Andaluz de Ciencias de la Tierra, CSIC-Universidad de Granada, E-18071 Granada, Spain, DIEGO GONZALEZ, Istituto IMM-CNR, Area della Ricerca CNR di Bologna, I-40129, Bologna, Italy, ORESTE PIRO, Department de Fisica, Universitat de les Illes Balears, E-07071 Palma de Mallorca, Spain — Some time ago two apparently dissimilar presentations were given at the 2007 Helmholtz Workshop in Berlin. One by J. Douthett and R. Krantz focused on the commonality between the mathematical descriptions of musical scales and the long-ranged, one-dimensional, anti-ferromagnetic Ising model of statistical physics. The other by J. Cartwright, D. Gonzalez, and O. Piro articulated a nonlinear dynamical model of pitch perception. Both approaches lead to a Farey series devil’s staircase structure. In the first case, the ground state magnetic phase diagram of the Ising model is a Farey series devil’s staircase. In the second case, the ear is modeled as a nonlinear system leading to a three-frequency resonant pitch perception model of the auditory system that exhibits a devil’s staircase phase-locked structure. In this poster we present a summary of each of these works side-by-side to illuminate the link between these two seemingly disparate systems. Adapted from JMM Vol. 4, No. 1, 57, Mar. 2010.

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