Quantifying and Classifying Nonlinearities in Mathematical Models

ALEXANDER SHUMWAY, MARK TRANSTRUM, Brigham Young Univ - Provo — Mathematical models, such as those used within physics, can be interpreted simply as mappings from a “parameter space” to predictions in a “data space.” It’s therefore natural to interpret a generic model as a manifold of possible predictions embedded in the space of data. Using the language of differential geometry, we can therefore understand and compare properties of models from a variety of disciplines in a single, unified language. Most interesting models are nonlinear, and understanding that nonlinearity is crucial for understanding the physical properties of the systems being modeled. Information about nonlinearity is contained in the three-index array of second derivatives of model predictions with respect to parameters. I discuss using a multilinear singular value decomposition to quantify different types of nonlinearity contained in this array and give examples from a variety of models.