Orbital Precession due to Central-Force Perturbations

GREGORY ADKINS, JORDAN MCDONNELL, Franklin and Marshall College — We calculate the precession of Keplerian orbits under the influence of arbitrary central-force perturbations. Our result is in the form of a one-dimensional integral that is straightforward to evaluate numerically. We demonstrate the effectiveness of our formula for the case of the Yukawa potential. We obtain analytic results for potentials of the form $V(r) = \alpha r^n$ and $V(r) = \alpha \ln(r/\lambda)$ in terms of the hypergeometric function ${}_2F_1 \left( \frac{1}{2} - \frac{n}{2}, 1 - \frac{n}{2}; 2; e^2 \right)$, where $e$ is the eccentricity. Our results reproduce the known general relativistic ($n = -3$) and cosmological constant ($n = 2$) precession formulas. Planetary precessions are often used to constrain the sizes of hypothetical new weak forces—our results allow for more precise, and often stronger, constraints on such proposed new forces.

Gregory Adkins
Franklin and Marshall College

Date submitted: 11 Jan 2007

Electronic form version 1.4