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Ghostbusting: Reviving quantum theories that were thought to be dead[1]

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The average quantum physicist on the street believes that a quantum-mechanical Hamiltonian must be Dirac Hermitian (symmetric under combined matrix transposition and complex conjugation) in order to be sure that the energy eigenvalues are real and that time evolution is unitary. However, the Hamiltonian $H = p^2 + ix^3$, for example, which is clearly not Dirac Hermitian, has a real positive discrete spectrum and generates unitary time evolution, and thus it defines a perfectly acceptable quantum mechanics. Evidently, the axiom of Dirac Hermiticity is too restrictive. While the Hamiltonian $H = p^2 + ix^3$ is not Dirac Hermitian, it is PT symmetric; that is, it is symmetric under combined space reflection P and time reversal T. In general, if a Hamiltonian H is not Dirac Hermitian but exhibits an unbroken PT symmetry, there is a procedure for determining the adjoint operation under which H is Hermitian. (It is wrong to assume a priori that the adjoint operation that interchanges bra vectors and ket vectors in the Hilbert space of states is the Dirac adjoint. This would be like assuming a priori what the metric $g^{\mu\nu}$ in curved space is before solving Einstein's equations.) In the past a number of interesting quantum theories, such as the Lee model and the Pais-Uhlenbeck model, were abandoned because they were thought to have an incurable disease. The symptom of the disease was the appearance of ghost states (states of negative norm). The cause of the disease is that the Hamiltonians for these models were inappropriately treated as if they were Dirac Hermitian. The disease can be cured because the Hamiltonians for these models are PT symmetric, and one can calculate exactly and in closed form the appropriate adjoint operation under which each Hamiltonian is Hermitian. When this is done, one can see immediately that there are no ghost states and that these models are fully acceptable quantum theories.

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