Einstein Prize Talk: Light-Cones in Relativity: Real, Complex and Virtual - with Applications

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We present some observations about certain unusual geometric structures that appear in both Minkowski space and asymptotically flat space-times. Three different types of light-cones are considered: ordinary real light-cones in Minkowski space, $M$, complex light-cones in the complexified Minkowski space, $M_C$, (Minkowski coordinates $x^a$ go to complex $z^a$) and third, virtual light-cones in asymptotically flat space-times. All three types are defined at future null infinity, $I^+$, ($I^+$ defined by the endpoints of infinite extensions of future directed null geodesics) via the vanishing of the shear of the null geodesics lying in the null surface. The virtual light-cones appear to converge to points in an auxiliary virtual space, H-space. Cones are labeled by their apex coordinate $x^a$ or $z^a$. Two applications are discussed. The first begins with asymptotically flat Maxwell fields written as $W=E+iB$. On each light cone, with apex $x^a$, extracting the $l=1$ harmonic of the Maxwell field determines the complex electromagnetic dipole moment, $D_{EM}=D_E+iD_M$. $D_{EM}$, a function of $x^a$, can be analytically extending into $M_C$. Its zero set, points in $M_C$ where $D_{EM}(z^a)$ vanishes, is a complex curve called the complex center of charge world-line. The second application virtually repeats the Maxwell case but now for asymptotically flat Einstein/Einstein-Maxwell fields. In the asymptotic region of each virtual light-cone, extracting the $l=1$ harmonics from the asymptotic gravitational field (the Weyl tensor) yields the complex gravitational dipole, $D_{Grav}=D_{Mass}+iD_{Spin}$. Each cone is labeled by its H-space apex $z^a$. $D_{Grav}(z^a)$ is thus a function on H-space. Its zero set determines an H-space curve: the complex center of mass world-line. Interior space-time physical quantities and dynamics, (e.g. center of mass, spin, angular momentum, linear momentum, force, eqs. of motion) are identified at $I^+$ and described in terms of this complex world-line.