Transverse Gravitational Redshift ALEX MAYER, Jay Pritzker Fellowship — These two independently derived equations predict a relativistic redshift between two ideal clocks at identical gravitational potential separated by a constant distance ($d$). The general formula (1) includes an elliptic integral of the second kind ($E$ |2). Here, $b$ is the effective radius of the Earth ($\sim 6371$ km) and $d$ the horizontal distance between two clocks at the altitude $b$. Eq. (2) is similar to the approximate formula $z \approx gh/c^2$ for the radial Einstein shift as both formulas are valid for small separation distances and both are readily derived from first principles. For a clock separation of 500 km at sea level, the two formulas yield the same prediction ($z \sim 10^{-12}$) to an accuracy of $\pm 1 \times 10^{-15}$. For 50 km, these formulas yield the same prediction ($z \sim 10^{-14}$) to an accuracy of $\pm 1 \times 10^{-19}$. This empirical prediction can be tested using modern atomic clocks and frequency transfer by fiber-optic cable.

$$z = \sec \left( \sqrt{\frac{8GM \cos \phi}{bc^2}} E \left( \frac{\phi}{2} \right) \right)^{\phi} - \phi - \frac{\phi}{2} \left[ \phi = \frac{d}{2b} \right]$$ (1)

$$z = \frac{\Delta f}{f} \approx \frac{4GM}{bc^2} \sin^2 \left( \frac{d}{2b} \right) \left[ d \ll b \right]$$ (2)