Transverse Gravitational Redshift

ALEX MAYER, Jay Pritzker Fellowship — These two independently derived equations predict a relativistic redshift between two ideal clocks at identical gravitational potential separated by a constant distance \(d\). The general formula (1) includes an elliptic integral of the second kind \(E | 2\). Here, \(b\) is the effective radius of the Earth \((\sim 6371 km)\) and \(d\) the horizontal distance between two clocks at the altitude \(b\). Eq. (2) is similar to the approximate formula \(z \approx gh/c^2\) for the radial Einstein shift as both formulas are valid for small separation distances and both are readily derived from first principles. For a clock separation of 500 \(km\) at sea level, the two formulas yield the same prediction \((z \sim 10^{-12})\) to an accuracy of \(\pm 1 \times 10^{-15}\). For 50 \(km\), these formulas yield the same prediction \((z \sim 10^{-14})\) to an accuracy of \(\pm 1 \times 10^{-19}\). This empirical prediction can be tested using modern atomic clocks and frequency transfer by fiber-optic cable.

\[
z = \sec \left( \frac{\sqrt{8GM\cos \phi}}{bc^2} E \left( \frac{\phi}{2} \right) \left[ \phi = \frac{d}{2b} \right] \right) - 1 \tag{1}
\]

\[
z = \frac{\Delta f}{f} \approx \frac{4GM}{bc^2} \sin^2 \left( \frac{d}{2b} \right) \tag{2}
\]

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