Dynamic and Thermodynamic Stability of Black Holes

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I describe recent work with Stefan Hollands that establishes a new criterion for the dynamical stability of black holes in $D \geq 4$ spacetime dimensions in general relativity with respect to axisymmetric perturbations: Dynamical stability is equivalent to the positivity of the canonical energy, $E$, on a subspace of linearized solutions that have vanishing linearized ADM mass, momentum, and angular momentum at infinity and satisfy certain gauge conditions at the horizon. We further show that $E$ is related to the second order variations of mass, angular momentum, and horizon area by $E = \delta^\mathcal{E} M - \sum \delta^\mathcal{E} J_i - (\kappa/\sqrt{\pi}) \delta^\mathcal{E} A$, thereby establishing a close connection between dynamical stability and thermodynamic stability. Thermodynamic instability of a family of black holes need not imply dynamical instability because the perturbations towards other members of the family will not, in general, have vanishing linearized ADM mass and/or angular momentum. However, we prove that all black branes corresponding to thermodynamically unstable black holes are dynamically unstable, as conjectured by Gubser and Mitra. We also prove that positivity of $E$ is equivalent to the satisfaction of a “local Penrose inequality,” thus showing that satisfaction of this local Penrose inequality is necessary and sufficient for dynamical stability.