The Geometry of Imaginary Length in Maxwell’s Equations and Relativity

FELIX T. SMITH, retired — Since the beginnings of relativity the reason for the imaginary time-dependent component in its \((x, y, z, ict)\) 4-space and in Minkowski’s space-time has been a mystery. The investigation of an unresolved issue in the structure of Maxwell’s equations leads unexpectedly to a recognition that the central imaginary quantity is a length and not a time. The geometry of this Maxwellian system is found to be both minutely time dependent with the Hubble expansion and Lobachevskian with a negative curvature. Because curvature is an inverse squared length, \(K_{\text{curv}} = R_{\text{curv}}^{-2}\), this negative curvature of the non-Euclidean geometry creates a generalized curvature length that is imaginary. It is the combination of global expansion with this negative curvature \(K_{\text{curv}}^{\text{Lob}} < 0\) that results in a curvature length that is both imaginary and increases with time, \(dR_{\text{curv}}(t) = icdt\). The imaginary is thus associated primarily with geometric concepts of length and curvature, connected only secondarily with time because of the expansion. Minkowski’s space-time associated the imaginary signature entirely with time and not with length, and cannot be sustained.