One-Dimensional Random Walks with One-Step Memory

KEVIN PIASKOWSKI, MICHAEL NOLAN, Millersville University — Formalized studies of random walks have been done dating back to the early 20th century. Since then, well-defined conclusions have been drawn, specifically in the case of one and two-dimensional random walks. An important theorem was formulated by George Polya in 1912. He stated that for a one or two-dimensional lattice random walk with infinite number of steps, \( N \), the probability that the walker will return to its point of origin is unity. The work done in this particular research explores Polya’s theorem for one-dimensional random walks that are non-isotropic and have the property of one-step memory, i.e. the probability of moving in any direction is non-symmetric and dependent on the previous step. The key mathematical construct used in this research is that of a generating function. This helps compute the return probability for an infinite \( N \). An explicit form of the generating function was devised and used to calculate return probabilities for finite \( N \). Return probabilities for various memory parameters were explored analytically and via simulations. Currently, further analysis is being done to try and find a relationship between memory parameters and number of steps, \( N \).