Special Relativity in Space-Time and Gravity in its Tangent Space Energy-Momentum

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— When two observers look at the same thing they measure things uniquely. A relativity theory tells us how we do a calculation that both observers agree upon. Special relativity applies to space-time. All inertial observers agree on the speed of light. The zero interval forms the light cone whose intersection defines the origin. All inertial observers can calculate the interval between any pair events and agree:

\[ t^2 - R^2 = t'^2 - R'^2 = \gamma^2 \]

which form parabolas. The Lorentz group characterizes the symmetry. All the changes for gravity happen in the phase space of space-time governed by the Poincar group. A phase space of space-time can be converted into an energy-momentum graph. Energy and angular momentum are conserved. The zeroes in energy-angular momentum space are the horizontal and vertical axes. My hypothesis is that non-inertial observers in a gravitational field will agree on the product of energy and angular momentum which forms hyperbolas but at 45 degrees to those in special relativity. A transformation law can be derived showing that

\[ E' = E/\gamma = m\frac{dt}{d\tau}/\gamma, P' = \gamma P = \gamma m\frac{dR}{d\tau}. \]

The Newtonian gravitational escape velocity for the gamma is consistent with light bending tests but not higher order experiments.

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