Rearranging the Exponential Wall for Large N-Body Systems\textsuperscript{1}

MARTIN DUNN, DEBORAH WATSON, University of Oklahoma — The general quantum mechanical $N$-body problem is widely believed to be $NP$ complete, a complexity class for which no polynomial time algorithm has been found. The resources required for an exact solution are thought to scale exponentially with $N$, doubling for every particle added. With current numerical resources, this problem “hits a wall” around $N = 10$ (within a factor of 2). We have formulated a perturbation method for bosons which uses symmetry to rearrange this exponential wall by shifting the work from numerical effort for a single $N$ to analytic work valid for all $N$. This series is invariant under the $N!$ operations of the symmetric group $S_N$, allowing group theory and graphical techniques to be used to solve the problem exactly and analytically at each order for an arbitrary interaction and for arbitrary $N$, i.e. the problem scales as $N^0$ at each order. The current work investigates the growth of complexity as a function of order by enumerating the graphs that correspond to the basis tensors at each order. The exponential complexity reappears in an exponential wall that scales with the order of the series. Thus, exact analytical calculations are possible for very large systems through low order.

\textsuperscript{1}This work was supported by ARO.