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Sommerfeld's Geometric View of Relativistic Vector Spaces: A Neglected Insight Revived FELIX T. SMITH, Retired — In 1909 Sommerfeld showed that the structure of a space-time 4-vector $(\Delta\mathbf{r}, ic\Delta t)$ implies that relativistic velocity vectors $\mathbf{v}$ occupy the space of geodesics on the 3-surface of a hypersphere of imaginary radius $S_{\text{vel}} = ic$. This peculiar-seeming result in relativistic 4-space is comparable to the existence of a real sphere within a Euclidean 3-space, and Sommerfeld recognized it later (1931) as implying a spherical 3-space of negative-curvature Lobachevsky geometry within the flat relativistic 4-space. These deductions are mathematically rigorous, and the implications for velocity vectors were accepted and used by Einstein (1921), but they have generally been ignored. The insights offered by Sommerfeld's geometric view are complementary to what the prevailing tensor techniques can do. I show how they can be readily extended and applied, developing new insights and results, including (a) a hyperbolic polar representation of velocity 4-vectors, (b) the equivalent representation for position-time 4-vectors, (c) a connection like that of a sphere with a tangent plane, joining relativistic 4-vectors and their nonrelativistic equivalents, providing quantitative connection formulas.

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