Universality and scaling in the $N$-body sector of Efimov physics

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In this talk I will illustrate the universal behavior that we have found inside the window of Efimov physics for systems made of $N \leq 6$ particles [1]. We have solved the Schrödinger equation of the few-body systems using different potentials, and we have changed the potential parameters in such a way to explore a range of two-body scattering length, $a$, around the unitary limit, $|a| \to \infty$. The ground- ($E^0_N$) and excited-state ($E^1_N$) energies have been analyzed by means of a recent-developed method which allows to remove finite-range effects [2]. In this way we show that the calculated ground- and excited-state energies collapse over the same universal curve obtained in the zero-range three-body systems. Universality and scaling are reminiscent of critical phenomena; in that framework, the critical point is mapped onto a fixed point of the Renormalization Group (RG) where the system displays scale-invariant (SI) symmetry. A consequence of SI symmetry is the scaling of the observables: for different materials, in the same class of universality, a selected observable can be represented as a function of the control parameter and, provided that both the observable and the control parameter are scaled by some material-dependent factor, all representations collapse onto a single universal curve. Efimov physics is a more recent example of universality, but in this case the physics is governed by a limit cycle on the RG flow with the emergence of a discrete scale invariance (DSI). The scaling of the few-body energies can be interpreted as follow: few-body systems (at least up to $N = 6$), inside the Efimov window, belong to the same class of universality, which is governed by the limit cycle. These results can be summarized by the following formula

$$E^0_N/E_2 = \tan^2 \xi, \quad \kappa^n_{AB} + \Gamma^N_n = \frac{e^{-\Delta(\xi)/2s_0}}{\cos \xi}. \quad (1)$$

where the function $\Delta(\xi)$ is universal and it is determined by the three-body physics, and $s_0 = 1.00624$. The parameter $\kappa^N_n$ appears as a scale parameter and the shift $\Gamma^N_n$ is a finite-range scale parameter introduced to take into account finite-range corrections [2].