Abstract Submitted
for the DFD05 Meeting of
The American Physical Society

Viscosity stratification and the aspect ratio of convection rolls
S.J.S. MORRIS, U. C. Berkeley — To clarify a mechanism by which earth's low–viscosity layer may increase the wavelength of mantle convection cells, we analyse the clockwise isothermal cellular motion driven by a uniform shear stress of magnitude τ applied at each end of a rectangle of height $2D$ and length L. The viscosity μ is a given piecewise-constant function of depth; within a low–viscosity channel of thickness d located at the top of the layer, $\mu = m\mu_1$; elsewhere, within the 'core', $\mu = \mu_1$. We show that in the double limit $d/D \to 0$, $m \to 0$, this two–layer flow is equivalent to one in single layer of viscosity μ_1 with a new boundary condition at its top representing the interaction of the channel and core flows. Let $x = x_*/L$, $y = y_*/D$ and $\psi = \mu_1\psi_*/\tau D^2$. Then the stream function ψ for the core motion satisfies the b.v.p. $\psi_{yyyy} + 2\alpha^2\psi_{xxyy} + \alpha^4\psi_{xxxx} = 0$; at $|x| = 1$, $\psi = 0$, $\alpha^2\psi_{xx} = -1$; at $y = 0$, $\psi = 0 = \psi_{yy}$; at $y = 1$, $\psi_{yy} - \alpha^2\psi_{xx} = 0$, and $\psi_{yyy} + 3\alpha^2\psi_{yxx} = 3\varepsilon\psi$. Here $\alpha = D/L$ and $\varepsilon = mD^3/d^3$. We find that for $\varepsilon \to 0$, the motion has two horizontal scales, namely D and $L_1 = D/\varepsilon^{1/2} \gg D$. If the rectangle length $L \sim L_1$, fluid sinks at one end and rises at the other; those end flows occur on the scale D, and are connected by a long–wave flow on the scale L_1. The cellular motion is closed within the low–viscosity layer. We have extended this method to treat convection rolls in a fluid of infinite Prandtl number. Our predicted heat flows agree well with those found in numerical simulations by Lenardic, Richards & Busse *et al* (2005) (*J. Geophys. Res.*, to appear).

S. J. S. Morris
U. C. Berkeley

Date submitted: 29 Jul 2005 Electronic form version 1.4