Leaky waves in boundary layer flow

JAN PRALITS, PAOLO LU-CHINI, DIMEC, University of Salerno, Italy — Linear stability analysis of boundary layer flow is traditionally performed by solving the Orr-Sommerfeld equation (OSE), either in a temporal or a spatial framework. The mode structure of the OSE is in both cases composed of a finite number of discrete modes which decay at infinity in the wall-normal direction $y$, and a continuous spectrum of propagating modes behaving as $\exp(\pm iky)$ when $y \to \infty$, with real $k$. A peculiarity of this structure is that the number of discrete modes changes with the Reynolds number, $Re$. They indeed seem to disappear behind the continuous spectrum at certain $Re$. This phenomenon is here investigated by studying the response of the Blasius boundary layer forced instantaneously in space and time. Since the solution of the forced and homogeneous Laplace-transformed problem both depend on the free-stream boundary conditions, it is shown here that a suitable change of variables can remove the branch cut in the Laplace plane. As a result, integration of the inverse Laplace transform along the two sides of the branch cut, which gives rise to the continuous spectrum, can be replaced by a sum of residues corresponding to an additional set of discrete eigenvalues. These new modes grow at infinity in the $y$ direction, and are analogous to the leaky waves found in the theory of optical waveguides, i.e. optical fibers, which are attenuated in the direction of the waveguide but grow unbounded in the direction perpendicular to it.

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