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Bounds on the enstrophy growth rate for solutions of the 3-d Navier-Stokes equations[1] LU LU, Department of Mathematics, University of Michigan, CHARLES DOERING, Department of Mathematics and Michigan Center for Theoretical Physics, University of Michigan — It is still an open problem whether smooth solutions to the 3-d Navier-Stokes equations lose regularity in finite time. But it is known that if enstrophy ($\|\omega\|^{\mathbf{2}}$) remains finite, the solution is regular. The growth rate of enstrophy can be estimated from the Navier-Stokes equations by Sobelev inequalities. In general form, $d\|\omega\|^{\mathbf{2}}/\mathbf{dt} \leq \mathbf{c}(\|\omega\|^{\mathbf{2}})^{\alpha}$, where c is a constant. In 2d, the exponent α is 2 and leads to regularity. However, $\alpha = 3$ in 3d, which shows only finite-time regularity of the solutions. In these types of estimates, incompressibility is not used. We formulate the search for the maximal enstrophy growth rate as a variational problem and include incompessibility as a constraint. The variational problem is solved numerically by a gradient-flow type algorithm. Our preliminary results show that $\alpha \approx 1.75$, which hints that solutions of the 3-d Navier-Stokes equations are regular for all time.

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