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Multiscale mixing efficiencies for passive scalars[1] JEAN-LUC THIFFEAULT, Imperial College, TIFFANY A. SHAW, University of Toronto, CHARLES R. DOERING[2], University of Michigan — The evolution of a passive scalar $\theta(\mathbf{x}, t)$ maintained by a steady spatially mean-zero source $s(\mathbf{x})$ and stirred by a prescribed velocity field $u(\mathbf{x}, t)$ is the advection-diffusion equation $\partial_t\theta + \mathbf{u} \cdot \nabla\theta = \kappa\Delta\theta + s$, where κ is the molecular diffusivity. In a finite volume the space-time averaged variances $\langle|\nabla\theta|^2\rangle$, $\langle\theta^2\rangle$ and $\langle|\nabla^{-1}\theta|^2\rangle$ are quantitative measures of the mixing due to the combination of advection and diffusion at small, intermediate and large scales respectively: the smaller the variances, the better the mixing. The enhancement of mixing over molecular diffusion due to the stirring is naturally gauged in dimensionless form by the small, intermediate and large scale mixing efficiencies $M_p^2 := \langle|\nabla^p\theta_0|^2\rangle \,/\, \langle|\nabla^p\theta|^2\rangle$ for $p \in \{-1, 0, 1\}$ where θ_0 is the steady solution of the diffusion equation $\partial_t\theta_0 = \kappa\Delta\theta_0 + s$ without stirring. These efficiencies depend on the structure of the source and the stirring as well as the domain and κ. We report rigorous mathematical limits on M_p in terms of the Péclet number $Pe = UL/\kappa$, where $U^2 = \langle|\mathbf{u}|^2\rangle$ and L is a domain length scale. The mathematical estimates are compared to the results of direct numerical simulations for a model problem.

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