Stratification and wavenumber selection in the instability of sedimenting spheroids

DAVID SAINTILLAN, Department of Mechanical Engineering, Stanford University, ERIC S. G. SHAQFEH, Department of Chemical Engineering, Stanford University, ERIC DARVE, Department of Mechanical Engineering, Stanford University — A dilute suspension of spheroids settling under gravity is unstable to density fluctuations as a result of hydrodynamic interactions. Whereas the original stability analysis of Koch & Shaqfeh (JFM 1989) predicts a maximum growth rate at zero wavenumber, recent experiments (Metzger et al. PRL 2005) and simulations (Saintillan et al. JFM 2006) both suggest that the instability in fact selects a finite wavelength corresponding to a typical cluster size. To elucidate the mechanism for this wavenumber selection, we revisit the linear stability analysis of Koch & Shaqfeh by including the effects of a stable stratification and of hydrodynamic center-of-mass diffusion. The analysis shows that the growth rates for the concentration fluctuations are damped at low wavenumbers by stratification and at high wavenumbers by diffusion, resulting in a maximum growth rate at an intermediate wavenumber. A scaling for the dependence of the cluster size on the vertical density gradient and on the local volume fraction is inferred. To validate the theory, numerical simulations are performed in stratified suspensions, where good agreement is found with the proposed scaling. In initially homogeneous suspensions, we argue that the process leading to the wavenumber selection may be controlled by the formation of vertical density gradients over the course of sedimentation.

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