Energy of Relative Equilibria of Identical Point Vortices

HASSENM AREF\textsuperscript{2}, Fluid DTU, Technical University of Denmark — Analytical formulae are derived for the energy of simple relative equilibria of \( N \) identical point vortices such as the regular polygons, both open and centered, the collinear states, and the various analytically known configurations consisting of two and three nested regular polygons with or without a vortex at the center. The main result is the following: If the vortices are situated at \( z_1, z_2, \ldots, z_N \), and if \( P(z) = (z - z_1)(z - z_2)\ldots(z - z_N) \), then \( \prod_{\alpha,\beta=1}^{N} (z_\alpha - z_\beta) = \prod_{\alpha=1}^{N} P'(z_\alpha) \), where the prime indicates a derivative with respect to \( z \). The logarithm of the absolute value of the left hand side is, in essence, the kinetic energy of fluid motion associated with the vortex pattern. This formula is known in the theory of polynomial equations as the formula for the discriminant expressed as a symmetric function of the roots. Its application to vortex dynamics appears to be new.

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