A Convectively Filtered Regularization of Multi-Dimensional Burgers Equation

GREGORY NORGARD, KAMRAN MOHSENI, University of Colorado, Boulder — Multi-dimensional Burgers equation, \( u_t + u \cdot \nabla u = \nu \Delta u \), can be considered as a simplified model of fluid dynamics. By sharing the convective nonlinear terms, it exhibits characteristics similar to those in the Euler and Navier-Stokes equations, particularly shocks and turbulence. Shocks and turbulence can both be attributed to the accumulation of energy in the high frequency wave modes, caused by the nonlinear term \( u \cdot \nabla u \). Typically this energy cascade is halted by introducing viscosity, balancing the nonlinearity with dissipation. An alternative solution is replacing the convective velocity with a low pass filtered velocity, \( \bar{u} \). The filtering reduces the energy in the higher wave modes, reducing the rate of the energy cascade. This method has been shown to regularize shocks in one-dimensional inviscid Burgers, \( u_t + uu_x = 0 \). This research extends this result into multiple dimensions with the equation, \( u_t + \bar{u} \cdot \nabla u = 0 \). The existence and uniqueness of a continuously differentiable solution is proven for a general class of filters. This regularization is then compared and contrasted with viscous Burgers in areas such as constants of motion, energy decay, shock thickness, and spectral energy decompositions.

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