Lagrangian method for advection diffusion equations using Gaussian averaging in space

SOPHIE LOIRE, IGOR MEZIC, UCSB — Advection diffusion equations can be studied using a Lagrangian approach to analyze transport of densities. The motion of diffusive particles is given by the Langevin equation \( d\vec{X} = \vec{V} dt + \sqrt{2D} d\vec{W} \). The Feynmann-Kac formula establishes a link between the random paths of this stochastic process and the advection-diffusion equation. The solution can be written as an expectation with respect to the probability measure of the Wiener process. We applied this idea to a backward monte carlo method and compare it to a new method using gaussian averaging in space \( \frac{1}{\pi\sigma^2} \exp \left( \frac{|\vec{x} - \vec{x}_0|^2}{\sigma^2} \right) \). In the cases where the solution is desired on a lower dimensional subset of the domain, a backward method using Feynman-Kac formula can be very efficient. But if the grid in this subset is very fine the Feynmann-Kac method is limited by the fact that a high number of Wiener processes need to be simulated at each grid point then the gaussian averaging method becomes more efficient. We apply these methods to the study of electrokinetic device.

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