Velocity fluctuations and energy amplification in laminar fluid flows

JOSE M. ORTIZ DE ZARATE, Fisica Aplicada I. Universidad Complutense, 28040 Madrid, Spain, JAN V. SENGERS, IPST and Burgers Program for Fluid Dynamics, University of Maryland, College Park, Maryland 20742-8510, USA

We present a systematic procedure for evaluating the intrinsic velocity fluctuations and the resulting intrinsic energy amplification that are always present in laminar fluid flows. For this purpose we formulate a stochastic Orr-Sommerfeld equation and a stochastic Squire equation by applying a fluctuation-dissipation theorem for the random part of the dissipative stresses. From the solution of the stochastic Orr-Sommerfeld and Squire equations the intrinsic energy amplification can be deduced. As an illustration of the procedure we present an explicit solution for the case of planar Couette flow. We first solve the fluctuating hydrodynamics equations in the bulk, obtaining an exact representation of the spatial spectrum of the velocity fluctuations valid for large wave numbers. The resulting energy amplification is proportional to $Re^{3/2}$. Next, we show how to a good approximation confinement can be incorporated by a simple Galerkin projection technique. The effect of the boundary conditions is to reduce the energy amplification to a logarithmic dependence on $Re$.

We shall also indicate how an exact solution for the case of confined geometries can be obtained by an expansion into a set of hydrodynamic modes, conveniently expressed in terms of Airy functions.

Jose M. Ortiz de Zarate
Fisica Aplicada I. Universidad Complutense. 28040 Madrid, Spain

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