A topological approach to three-dimensional laminar mixing

MATTHEW FINN, NATHANIEL JEWELL, The University of Adelaide — Research into laminar mixing has enjoyed a renaissance in the last decade since the realisation that the Thurston–Nielsen (TN) theory of surface homeomorphisms can assist in designing efficient “topologically chaotic” mixers. However, published results to date have been limited to 2D flows and quasi-3D protocols. Motivated by a simple stretching and folding argument used to derive stretching bounds in 2D flows (what Thurston describes as the iterate-and-guess method for constructing invariant train-tracks), we propose a topological approach to fully 3D fluid mixing. We consider periodic braiding of fluid in two orthogonal directions by inducing a flow with strategically placed ghost rods. The action of this braiding may be encoded by a transition matrix describing how certain area elements are mapped onto each other. The spectral radius of this matrix then furnishes an estimate of large-time asymptotic area growth rate. While this approach to mixing does not sit within the rigorous setting for TN theory, we find nonetheless that the predicted area stretch rates are very sharp for some model flows. Furthermore, we find that certain braids that are topologically trivial in 2D are quite effective in 3D.

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