3D aspects of mixing and transport in tumbled granular flow
IVAN C. CHRISTOV, RICHARD M. LUEPTOW, JULIO M. OTTINO, Northwestern University, ROB STURMAN, University of Leeds, STEPHEN WIGGINS, University of Bristol — The exploration of the kinematic structures that emerge in 3D flows has only just begun (see, e.g., Focus on Fluids in JFM vol. 654, 2010). Tumbled granular flow in a spherical container rotated sequentially about two distinct axes is a convenient physical system in which to investigate these issues. The flow can “switch” between 2D motion (dynamics restricted to 2D manifolds) and fully-3D motion depending on the choice of angles of rotation about the axes. We compute explicitly the action-angle transformation, period along trajectories and exact location of normally-hyperbolic and elliptic period-one curves from the piecewise-defined nonlinear dynamical system. This provides the basis for a definition of a 3D notion of an “island.” These theoretical results also allow for the “optimization” of the angles of rotation in the protocol. An extensive numerical investigation of the “goodness of mixing” is performed using Danckwerts’ intensity of segregation $I$. By fitting the decay rate and asymptotic value of $I$, we can understand the effects of the protocol parameters and how mixing varies across the volume of the tumbler. Finally, we establish the existence of “adiabatic” structures (2D manifolds exhibiting chaotic and ergodic dynamics) and study the persistence of these barriers to radial transport as the flow is perturbed into the fully-3D regime.

Ivan C. Christov

Date submitted: 22 Jul 2010