An accurate and efficient method to compute steady uniform vortices

P. LUZZATTO-FEGIZ, C.H.K. WILLIAMSON, Cornell University — Steady uniform vortices represent a widely used approximation in a broad range of contexts, ranging from dynamics of plasmas to geophysical flows. Surprisingly, computing steady uniform vortices still presents several challenges, since vortex boundaries may develop high-curvature regions, which can be prohibitively expensive to resolve. Further to this, flows can bifurcate to lower-symmetry states, which may be difficult to compute reliably. Currently, one must choose between affordable relaxation methods and more reliable approaches based on Newton iteration. However, while the first cannot resolve flows without symmetry (Dritschel 1985), the second are unaffordable for vortices with high-curvature regions (Saffman & Szeto 1980; Elcrat et al. 2005). Hence it is typically impossible to compute a family of steady vortices in its entirety. We overcome these limitations by introducing a new discretization, based on an “inverse-velocity map”, which makes Newton iteration affordable for vortices with high-curvature boundaries. By employing our numerical method in conjunction with the IVI-diagram stability approach (LF&W PRL 2010), we explore the full bifurcation structure of several classical flows, including elliptical vortices, co-rotating and counterrotating vortex pairs, and vortex streets. We have also successfully employed our discretization for other fluid problems, such as steep gravity waves.

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