Variational Theory of Hyperbolic Lagrangian Coherent Structures

GEORGE HALLER, McGill University — We describe a mathematical theory that clarifies the relationship between Lagrangian Coherent Structures (LCS) in unsteady fluid flows and invariants of the Cauchy-Green strain tensor field. Motivated by physical observations of tracer patterns, we define hyperbolic LCS as material surfaces that extremize an appropriate finite-time normal repulsion or attraction measure. Solving this variational problem leads to computable sufficient and necessary criteria for LCS. We also discuss constrained LCS problems, as well as the robustness of LCS under numerical errors and data imperfection. In several examples, we show how these results resolve earlier inconsistencies in the theory of LCS.