On Lagrangian and vortex-surface fields in Taylor-Green and Kida-Pelz flows

DALE PULLIN, YUE YANG, California Institute of Technology

Abstract

— A methodology is developed for constructing smooth scalar fields $\phi$ for Taylor-Green and Kida-Pelz velocity fields that, at $t = 0$, satisfy $\omega \cdot \nabla \phi = 0$. We refer to such fields as vortex-surface fields. Iso-surfaces of $\phi$ then define vortex surfaces. Given the vorticity, our definition of a vortex-surface field is shown to admit nonuniqueness, and this is resolved numerically using an optimization approach. Equations describing the evolution of vortex-surface fields are obtained for both inviscid and viscous incompressible flows. For the former, the Helmholtz vorticity theorem shows that Lagrangian material surfaces which are vortex (or vorticity) surfaces at the initial time remain so for later times. By tracking $\phi$ as a Lagrangian field in slightly viscous flows, we show that the well-defined evolution of Lagrangian surfaces that are initially vortex surfaces can be a good approximation to vortex surfaces at later times prior to vortex reconnection. In the evolution of such Lagrangian fields, we observe that initially blob-like vortex surfaces are progressively stretched to sheet-like shapes, with subsequent rolling up of structures near the interface. The nonlocal geometry in the evolution is quantified by differential geometry properties.

$^1$Supported by the NSF.