A New Theory of Oscillating Flows

VLADIMIR VLADIMIROV, Mathematics, University of York, UK — A new theory of viscous oscillatory flows has been developed. Our theory represents an adaptation of the Vishik-Lyusternik approach combined with the two-timing and averaging methods. We consider the high Re viscous incompressible flows driven by a vibrating boundary for the simple geometry of a half-space. From the physical viewpoint the considered boundary conditions may be seen as the tangential vibrations of material points of a plane stretchable membrane. The main result is the obtaining of the general, global, and uniformly valid asymptotic solutions of the Navier-Stokes equations. These solutions satisfy general oscillating boundary conditions and three different settings of the scaling parameters (that correspond to the strong, moderate, and weak nonlinearities). We have derived that the streaming part of any solution is governed by either the full Navier-Stokes equations or Stokes’ equations (both with the unit Re) as well as by the precisely derived effective boundary conditions. The examples of the spatially periodic vibrations of the boundary and the angular torsional vibrations of an infinite rigid disc have been considered. In the sharp contrast to all previous theories of oscillating flows (see e.g. Batchelor’s “Introduction to fluid dynamics,” formula 5.13.15) our solutions do not deal with any secular (infinitely growing with the inner normal coordinate) terms. This new approach may be seen as a revolutionary step in the field, since for the very first time it does not use the asymptotic matching procedures and the boundary layer theories.