Instability of Point Vortex Leapfrogging

LAUST TOPHØJ, Department of Physics, Technical University of Denmark, HASSAN AREF, ESM Dept., Virginia Tech, and Center for Fluid Dynamics, Technical University of Denmark — The dynamics of interacting point vortices on the unbounded plane can be chaotic if the number of vortices is at least four. The chaotic dynamics is governed by the existence of unstable structures in the phase space, [Tophøj & Aref, Phys. Fluids, 20, 093605 (2008)]. Such structures may be hyperbolic fixed points of the dynamical system, or unstable periodic orbits. Chaos arises as the system is repeatedly repelled by these structures, bouncing back and forth between them. The leapfrogging motion of two vortex pairs possessing a common axis of symmetry is an example of an integrable periodic motion of a four-vortex system. The stability of this periodic motion has been studied numerically by Acheson [Eur. J. Phys. 21, 269 (2000)] whose results indicate instability for some but not all parameters. We discuss the stability of leapfrogging, using methods from Floquet theory. Analogies will be drawn to instabilities of the von Kármán vortex street that can cause the vortex street to break up into vortex pairs moving away from the central axis.

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