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Maximum Enstrophy Growth in Burgers Equation DIEGO AYALA, BARTOSZ PROTAS, Department of Mathematics and Statistics, McMaster University — The regularity of solutions of the Navier–Stokes equation is controlled by the boundedness of the enstrophy $\mathcal{E}$. The best estimate for its rate of growth is $d\mathcal{E}/\lceil\sqcup \leq \mathcal{C}\mathcal{E}^{\ni}$, for $C > 0$, leading to the possibility of a finite–time blow–up when straightforward time integration is used. Recent numerical evidence by Lu & Doering (2008) supports the sharpness of the instantaneous estimate. Thus, the central question is how to extend the instantaneous estimate to a finite–time estimate in a way that will incorporate the dynamics imposed by the PDE. We state the problem of saturation of finite–time estimates for the enstrophy growth as a PDE–constrained optimization problem, using the Burgers equation as a "toy model". The following problem is solved numerically:

$$\max_{\phi}[\mathcal{E}(\mathcal{T}) - \mathcal{E}(\prime)] \quad \text{subject to} \quad \mathcal{E}(\prime) = \mathcal{E}_\prime$$

where ϕ represents the initial data for Burgers equation, for a wide range of values of $T > 0$ and $\mathcal{E}_\prime$ finding that the maximum enstrophy growth in finite time scales as $\mathcal{E}_\prime^\alpha$ with $\alpha \approx 3/2$, an exponent different from $\alpha = 3$ obtained by analytic means.

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