Stability of icosahedral configurations of point vortices on a sphere

VITALII OSTROVSKYI, PAUL NEWTON, University of Southern California — Using icosahedron as the initial geometric distribution of point vortices on a sphere we show existence of icosahedral relative equilibrium configurations. To characterize these configurations we apply method based on finding the fixed points of the nonlinear dynamical system governing the $N(N-1)/2$ equations for interparticle distances. Obtained equations give sufficient conditions for the relative equilibria and lead to a problem of finding solutions to $A\vec{\Gamma} = 0$, where $\vec{\Gamma} \in \mathbb{R}^N$ is the vector of vortex strengths, and $A \in \mathbb{R}^{M \times N}$ is a rectangular, non-normal ($AA^T \neq A^T A$) ‘configuration’ matrix determined by the particle positions. Using singular value decomposition of $A$ we prove that for icosahedron the $\text{Nullspace}(A)$ is 7 dimensional. Vertex and edge stabilizers, as subgroups of icosahedral symmetry group, are used to build the set of symmetric icosahedral configurations with non-negative strengths. Using exact solution of equations of motion we prove stability of vortex pair configurations. Energy-momentum method is used to study stability of symmetric icosahedral relative equilibria. To prove instability of some of the configurations we show that the matrix of linearized system has eigenvalues with positive real parts. Using the stability results we build an example of linear superposition of stable configurations which gives unstable configuration.

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Date submitted: 16 Aug 2010