Optimal Mixing, Part II\textsuperscript{1} ZHI LIN, Institute for Mathematics and its Applications, JEAN-LUC THIFFEAULT, University of Wisconsin, CHARLES DOERING, University of Michigan — We investigate optimal incompressible stirring to mix an initially inhomogeneous distribution of diffusionless passive tracers. The $H^{-1}$ Sobolev norm is adopted as the quantitative mixing measure of the tracer concentration field: its vanishing as $t \to \infty$ is equivalent to the stirring flow’s mixing property in the sense of ergodic theory. We derive rigorous bounds on the rate of mixing by flows with fixed energy or energy dissipation rate constraints, and determine the flow field that instantaneously maximizes the decay of the mixing measure – when such a flow exists – by solving a variational problem. When no such ‘steepest descent’ flow exists (a possible but non-generic situation) we determine the flow that maximizes the growth rate of the scalar’s $H^{-1}$ norm’s decay rate. This optimal stirring strategy is implemented numerically on a benchmark problem and compared to the rigorous bounds as well as an optimal control approach using a restricted set of flows.

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