Streaming potential revisited

EHUD YARIV, ORY SCHNITZER, ITZCHAK FRANKEL, Technion — Streaming-potential phenomena refer to the generation of bulk electric fields by imposed relative motion between a charged solid and the Debye layer adjacent to it. Realistic scenarios are adequately described by the thin-Debye-layer limit \( \delta \to 0 \) (\( \delta \) denoting the dimensionless Debye thickness), which has been addressed by Cox (1997). Cox’s analysis has established that the perturbation to the flow, neglected in the earlier investigations, gives rise to an \( O(\delta^4) \) force that dominates that contributed by Maxwell stresses. Cox’s theory is founded upon the assumption of \( O(1) \) Hartmann and Péclet numbers. We demonstrate that the product of these numbers is actually \( O(\delta^{-2}) \) and accordingly revisit the generic problem of streaming-potential. Electric-current matching between the Debye layer and the bulk provides an inhomogeneous Neumann condition governing the electric field in the latter. This field, in turn, results in a velocity perturbation animated by a Smoluchowski-type slip condition. Owing to dominant convection, the present analysis yields an asymptotic structure considerably simpler than that of Cox (1997): the electro-viscous effect now already appears at \( O(\delta^2) \) and is contributed by both Maxwell and viscous stresses. The present paradigm is illustrated for the prototypic problem of a sphere sedimenting in an unbounded fluid, with the resulting drag correction differing from that calculated by Cox (1997).