Bilinear relative equilibria of identical point vortices\textsuperscript{1} HASSAN AREF, Virginia Tech, PETER BEELEN, MORTEN BRØNS, Technical University of Denmark — A new class of bilinear relative equilibria of identical point vortices in which the vortices are constrained to be on two perpendicular lines, taken to be the \(x\)- and \(y\)-axes of a cartesian coordinate system, is introduced and studied. In general we have \(m\) vortices on the \(y\)-axis and \(n\) on the \(x\)-axis. We define generating polynomials \(q(z)\) and \(p(z)\), respectively, for each set of vortices. A second order, linear ODE for \(p(z)\) given \(q(z)\) is derived. Several results relating the general solution of the ODE to relative equilibrium configurations are established. Our strongest result, obtained using Sturm’s comparison theorem, is that if \(p(z)\) satisfies the ODE for a given \(q(z)\) with its imaginary zeros symmetric relative to the \(x\)-axis, then it must have at least \(n - m + 2\) simple, real zeros. For \(m = 2\) this provides a complete characterization of all zeros, and we study this case in some detail. In particular, we show that given \(q(z) = z^2 + \eta^2\), where \(\eta\) is real, there is a unique \(p(z)\) of degree \(n\), and a unique value of \(\eta^2 = A_n\), such that the zeros of \(q(z)\) and \(p(z)\) form a relative equilibrium of \(n + 2\) point vortices. We show that \(A_n \approx \frac{2}{3}n + \frac{1}{2}\), as \(n \to \infty\), where the coefficient of \(n\) is determined analytically, the next order term numerically.

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