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Bilinear relative equilibria of identical point vortices[1] HASSAN AREF, Virginia Tech, PETER BEELEN, MORTEN BRØNS, Technical University of Denmark — A new class of bilinear relative equilibria of identical point vortices in which the vortices are constrained to be on two perpendicular lines, taken to be the x- and y-axes of a cartesian coordinate system, is introduced and studied. In general we have m vortices on the y-axis and n on the x- axis. We define generating polynomials $q(z)$ and $p(z)$, respectively, for each set of vortices. A second order, linear ODE for $p(z)$ given $q(z)$ is derived. Several results relating the general solution of the ODE to relative equilibrium configurations are established. Our strongest result, obtained using Sturm's comparison theorem, is that if $p(z)$ satisfies the ODE for a given $q(z)$ with its imaginary zeros symmetric relative to the x-axis, then it must have at least $n - m + 2$ simple, real zeros. For $m = 2$ this provides a complete characterization of all zeros, and we study this case in some detail. In particular, we show that given $q(z) = z^2 + \eta^2$, where η is real, there is a unique $p(z)$ of degree n, and a unique value of $\eta^2 = A_n$, such that the zeros of $q(z)$ and $p(z)$ form a relative equilibrium of $n + 2$ point vortices. We show that $A_n \approx \frac{2}{3}n + \frac{1}{2}$, as $n \to \infty$, where the coefficient of n is determined analytically, the next order term numerically.

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