Dynamical systems characterization of the poor man’s Navier–Stokes equations
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The Navier–Stokes (N.–S.) equations governing fluid flow consist of a system of time-dependent, multi-dimensional, non-linear partial differential equations (PDEs) which cannot be solved in real time using current, or near-term foreseeable, computing hardware. The poor man’s Navier–Stokes (PMNS) equations comprise a discrete dynamical system (DDS) that is algebraic—hence, easily (and rapidly) solved—and yet which retains many (possibly all) of the temporal behaviors of the full (PDE) N.–S. system at specific spatial locations. In this investigation we outline the derivation of the PMNS equations beginning with the incompressible N.–S. equations. We then consider common techniques to understand the DDS sensitivity to initial conditions (SIC) through calculation of bifurcation diagrams, Lyapunov exponents, and fractal dimension. These techniques are studied with consideration of their ease of computation, and ability to characterize and describe system behavior. The time series generated by the DDS are used to obtain power spectral densities (PSDs) which can be used to categorize most system behaviors. Some chaotic behaviors, however, can be difficult to distinguish via PSD analysis alone; thus we investigate the ability of other methods to characterize the system response.