High-Order Finite-Difference Solution of the Poisson Equation Involving Complex Geometries in Embedded Meshes

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The Poisson equation is of central importance in the description of fluid flows and other physical phenomena. In prior work, Marques, Nave, and Rosales introduced the Correction Function Method (CFM) to obtain fourth-order accurate solutions for the constant coefficient Poisson problem with prescribed jump conditions for the solution and its normal derivative across arbitrary interfaces. Here we combine this method with the ideas introduced by Mayo to solve other Poisson problems involving complex geometries. In summary, we are able to rewrite the problem as a boundary integral equation in terms of a potential distribution over the boundary or interface. The solution of this integral equation is discontinuous across the boundary or interface. Hence, after this integral equation is solved using standard techniques, the potential distribution can be used to determine the jump discontinuities. We are then able to use the CFM to solve the resulting Poisson equation with jump discontinuities. The outcome is a fourth-order accurate scheme to solve general Poisson problems which, over arbitrary geometries, has a cost that is approximately twice that of a fast Poisson solver using FFT on a rectangular geometry of the same size. Details of the method and applications will be presented.

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Date submitted: 05 Aug 2011