A high order solver for the unbounded Poisson equation with specific application to the equations of fluid kinematics

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