On Optimal Vortex Structures for Palinstrophy Generation
DIEGO AYALA, BARTOSZ PROTAS, McMaster University — We are interested in identifying the vortex structures which lead to the largest growth of palinstrophy $P$ in flows governed by the 2D Navier–Stokes equation in a periodic domain. This problem is a 2D counterpart of the problem concerning the maximal production of enstrophy $E$ in 3D Navier–Stokes flows which is inherently related to the question of finite–time singularity formation. We investigate the sharpness of the following analytic estimates

$$\frac{dP}{dt} \sim P^E \quad \text{and} \quad \max_{\|l\|>l} P(l) \sim P(t)^E \quad \text{as} \quad P \to \infty$$

by studying suitable optimization problems. These problems are solved for different values of the palinstrophy using modern methods of PDE–constrained optimization combined with DNS. In regard to the instantaneous problem, we discover two families of maximizing fields characterized by distinct properties. We also present evidence that the maximum growth of $\frac{dP}{dt}$ with $P$ is in fact weaker than suggested by the analytic estimate.

Diego Ayala
McMaster University

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