

Abstract Submitted  
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**Wall to Wall Optimal Transport**<sup>1</sup> GREGORY P. CHINI, University of New Hampshire, PEDRAM HASSANZADEH, Harvard University, CHARLES R. DOERING, University of Michigan — How much heat can be transported between impermeable fixed-temperature walls by incompressible flows with a given amount of kinetic energy or enstrophy? What do the optimal velocity fields look like? We employ variational calculus to address these questions in the context of steady 2D flows. The resulting nonlinear Euler–Lagrange equations are solved numerically, and in some cases analytically, to find the maximum possible Nusselt number  $Nu$  as a function of the Péclet number  $Pe$ , a measure of the flow’s energy or enstrophy. We find that in the fixed-energy problem  $Nu \sim Pe$ , while in the fixed-enstrophy problem  $Nu \sim Pe^{10/17}$ . In both cases, the optimal flow consists of an array of convection cells with aspect ratio  $\Gamma(Pe)$ . Interpreting our results in terms of the Rayleigh number  $Ra$  for relevant buoyancy-driven problems, we find  $Nu \leq 1 + 0.035Ra$  and  $\Gamma \sim Ra^{-1/2}$  for porous medium convection (which occurs with fixed energy), and  $Nu \leq 1 + 0.115Ra^{5/12}$  and  $\Gamma \sim Ra^{-1/4}$  for Rayleigh–Bénard convection (which occurs with fixed enstrophy and for free-slip walls).

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