A Second-Order Finite-Difference Scheme for the Lattice Boltzmann Method

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— The lattice Boltzmann method (LBM) is being increasingly used as an alternative solver for the isothermal Navier-Stokes equation, as well as for other complex flows. However, due to an innate coupling between the velocity and the configuration space, LBM is restricted to uniform grids. This is a serious impediment for simulating flows with large gradients, flow around objects, etc. The discrete Boltzmann-BGK equation, which forms the basis of LBM, can be viewed as a set of hyperbolic equations with constant coefficients and a source term. We, therefore, use the finite difference method to discretize the Boltzmann-BGK equation (FDLBM). In FDLBM, the velocity-lattice is uncoupled from the spatial lattice allowing us to choose discrete velocities and space-time steps independently. The currently available FDLBM models have either narrow a stability range, or have large computational costs. To overcome these constraints, we employ the Lax-Wendroff scheme for the advection part, and central-difference for the spatial gradients, resulting in a scheme that is both explicit and second-order in both space and time. The proposed scheme is validated for an isothermal incompressible lid-driven cavity flow. The results indicate improved stability (in terms of CFL conditions) compared to the current explicit FDLB models, due to the addition of the second-order temporal terms. The maximum Reynolds number that can be simulated stably is also much higher. The relationship between the discrete time-step and the relaxation parameter, and extension of the FDLBM to a non-uniform mesh are also discussed.