Extreme Vortex States and the Growth of Enstrophy in 3D Incompressible Flows DIEGO AYALA, McMaster University, University of Michigan, BARTOSZ PROTAS, McMaster University — In this investigation we analyze a family of extreme vortex states which maximize the instantaneous production of enstrophy under Navier-Stokes dynamics on 3D periodic domains. They are found by numerically solving suitably constrained optimization problems and include other well-known flows, such as the Taylor-Green vortex and the ABC flow, as special cases. Initially discovered by Lu & Doering (2008), these optimal vortex states saturate an analytic upper bound on the rate of growth of enstrophy, indicating that this estimate is in fact sharp. We provide a numerical characterization of the set of initial data for which smooth solutions are guaranteed to exist for all times, thereby offering a physical interpretation of a well-known result of mathematical analysis. The results from high-resolution direct numerical simulations indicate that the flows triggered by these optimal fields produce a larger finite-time growth of enstrophy than the flows obtained from other widely-used initial conditions, such as the Taylor-Green vortex, Lamb dipoles and perturbed anti-parallel vortex tubes. Although numerical in nature, these results illustrate a systematic approach to finding a worst-case initial condition which could lead to the potential formation of a singularity in finite-time.

Diego Ayala
McMaster University, University of Michigan

Date submitted: 30 Jul 2014
Electronic form version 1.4