A Missing Puzzle Piece in Murray’s Law: the Optimal Angle of Junctions

RUO-QIAN WANG, KATHERINE TAYLOR, AMOS G. WINTER, Massachusetts Inst of Tech-MIT, GLOBAL ENGINEERING AND RESEARCH LAB TEAM — Branching flows are common in biological systems, such as the circulatory and respiratory systems of animals. The optimal radii of parent and daughter branches can be explained with Murray’s law, which dictates that the sum of metabolic and pumping costs is minimized. Murray’s Law can be used to determine the diameter of cascading channels but misses an important parameter: the angles of the branches. Past hydraulic studies have investigated the angle effect, but have not focused on whether this geometry follows Murray’s Law; while a simple network optimization is able to show that at low Reynolds numbers a branch with a parent channel connecting to \( n \) equally distant channels obeying Murray’s Law has a minimum total head loss with a branching angle \( \theta \), such that \( \cos \theta = n^{-\frac{2}{3}} \), but it’s not valid for high Reynolds number flows, which may experience separation and turbulence at the branches. The present study is focused on determining the optimal branch angle that complies with Murray’s Law for moderate Reynolds numbers. Computational studies using Open FOAM and experiments using 3D printed branched channels will be presented. These results will be used to quantify the effect of Reynolds number on optimal branch geometry.

Ruo-Qian Wang
Massachusetts Inst of Tech-MIT

Date submitted: 01 Aug 2014

Electronic form version 1.4