Optimum heat transport by coherent Rayleigh-Bénard convection

FABIAN WALEFFE, ANAKEWIT BOONKASAME, LESLIE SMITH, University of Wisconsin-Madison — A classic marginal boundary layer argument suggests that the heat transport $Nu$ scales like $Ra^{1/3}$ while upper bound theories give that $Nu$ is at most $\sim Ra^{1/2}$, where the Rayleigh number $Ra$ is the main control parameter for Rayleigh-Bénard convection. Turbulent data have shown various scalings between $Nu \sim Ra^{2/7}$ and $Ra^{1/3}$, depending on domain aspect ratio and various corrections. Here, we investigate coherent solutions of the Boussineq equations for the Rayleigh-Bénard problem with no-slip boundary conditions and Prandtl number 7. The primary solution that bifurcates from the conduction state at $Ra \approx 1708$ has been calculated up to $Ra \approx 4 \times 10^6$ and shows $Nu \sim Ra^{1/4}$ with a delicate spiral structure. A related solution that maximizes $Nu$, at least locally, has been calculated up to $Ra = 10^9$ and it scales as $Nu - 1 \sim 0.12 Ra^{0.31}$ for $10^7 < Ra < 10^9$, quite similar to turbulent data. This is a simple yet multi-scale coherent solution whose horizontal wavelength is $\sim Ra^{-0.22}$ in that range. It is unstable to larger scale perturbations and in particular to mean flows, yet it appears to be relevant as a backbone for turbulent solutions, in particular, setting the scale and strength of elemental plumes.

Fabian Waleffe
University of Wisconsin-Madison

Date submitted: 01 Aug 2014  Electronic form version 1.4