The importance of being fractional in mixing: optimal choice of the index $s$ in $H^{-s}$ norm\(^1\) LUKAS VERMACH, Cambridge Centre for Analysis, DAMTP, University of Cambridge, C.P. CAULFIELD, BPI & DAMTP, University of Cambridge — A natural measure of homogeneity of a mixture is the variance of the concentration field, which in the case of a zero-mean field is the $L^2$-norm. Mathew et al. (Physica D, 2005) introduced a new multi-scale measure to quantify mixing referred to as the mix-norm, which is equivalent to the $H^{-1/2}$ norm, the Sobolev norm of negative fractional index. Unlike the $L^2$-norm, the mix-norm is not conserved by the advection equation and thus captures mixing even in the non-diffusive systems. Furthermore, the mix-norm is consistent with the ergodic definition of mixing and Lin et al. (JFM, 2011) showed that this property extends to any norm from the class $H^{-s}$, $s > 0$. We consider a zero-mean passive scalar field organised into two layers of different concentrations advected by a flow field in a torus. We solve two non-linear optimisation problems. We identify the optimal initial perturbation of the velocity field with given initial energy as well as the optimal forcing with given total action (the time integral of the kinetic energy of the flow) which both yield maximal mixing by a target time horizon. We analyse sensitivity of the results with respect to $s$-variation and thus address the importance of the choice of the fractional index

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