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The importance of being fractional in mixing: optimal choice of the index s in H^{-s} norm¹ LUKAS VERMACH, Cambridge Centre for Analysis, DAMTP, University of Cambridge, C.P. CAULFIELD, BPI & DAMTP, University of Cambridge — A natural measure of homogeneity of a mixture is the variance of the concentration field, which in the case of a zero-mean field is the L^2 -norm. Mathew *et.al.* (Physica D, 2005) introduced a new multi-scale measure to quantify mixing referred to as the mix-norm, which is equivalent to the $H^{-1/2}$ norm, the Sobolev norm of negative fractional index. Unlike the L^2 -norm, the mix-norm is not conserved by the advection equation and thus captures mixing even in the non-diffusive systems. Furthermore, the mix-norm is consistent with the ergodic definition of mixing and Lin *et al.* (JFM, 2011) showed that this property extends to any norm from the class H^{-s} , $s > 0$. We consider a zero-mean passive scalar field organised into two layers of different concentrations advected by a flow field in a torus. We solve two non-linear optimisation problems. We identify the optimal *initial perturbation* of the velocity field with given initial energy as well as the *optimal forcing* with given total action (the time integral of the kinetic energy of the flow) which both yield maximal mixing by a target time horizon. We analyse sensitivity of the results with respect to s -variation and thus address the importance of the choice of the fractional index

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