A Mathematical Proof of the Vortex Shedding Mechanism\textsuperscript{1}

MICHAEL BOGHOSIAN, KEVIN CASSEL, Illinois Inst of Tech — A novel mechanism leading to vortex splitting and subsequent shedding that is valid for both inviscid or viscous flows and external, internal, or wall-bounded flows is described. The mechanism, termed the Vortex-Shedding Mechanism (VSM), is simple and intuitive, requiring only two coincident conditions in the flow: (1) the existence of a location with zero momentum and (2) the presence of a net force having a positive divergence. Previous simulations of various flows have demonstrated the VSM numerically. Here, we present a mathematical proof of the VSM that is shown to be both a necessary and sufficient condition for a vortex splitting event in any two-dimensional, incompressible flow. The proof includes relating the positive divergence of the net force, condition (2) above, with the second invariant of the velocity gradient tensor, i.e. the Q-criterion. It is shown that the Q-criterion is identical to the determinant of the Hessian matrix for the streamfunction. As a result, the second-partial-derivative test on this Hessian matrix can provide a qualitative description on the behavior of the streamfunction, and thus vortices or recirculation regions, near critical points.

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