Solutions to the Navier-Stokes Equation in the complex plane

JONATHAN MESTEL, FLORENCIA BOSHIER, Imperial College London — A Stokes series is a theoretically attractive approach to solving the Navier-Stokes equations. Essentially the solution is expressed as a power series in the Reynolds number, $R_e$. At each order, a linear problem needs to be solved, providing a series representation of the solution for all $R_e$. This method was pioneered by Van Dyke in the 1970s. However, typically this series has a finite radius of convergence, and the solution has singularities at complex values of $R_e$. The behaviour of the series can be enhanced using a generalised Padé approximant technique. This method also predicts complex solution branches, and identifies bifurcation points to multiple solutions. Solutions branches for complex $R_e$ can be followed back onto the real $R_e$-axis. It is shown that in general the Navier-Stokes equations have more than one complex solution even for low (real) $R_e$. The intricate structure of complex solutions is followed in detail for Dean flows, and new branches are presented.