Extension to nonlinear stability theory of the circular Couette flow

PUN WONG YAU, SHIXIAO WANG, Auckland University, ZVI RUSAK, Rensselaer Polytechnic Institute — A nonlinear stability analysis of the viscous circular Couette flow to axisymmetric perturbations under axial periodic boundary conditions is developed. The analysis is based on investigating the properties of a reduced Arnol’d energy-Casimir function $A_{rd}$ of Wang (2009). We show that all the inviscid flow effects as well as all the viscous-dependent terms related to the flow boundaries vanish. The evolution of $\Delta A_{rd}$ depends solely on the viscous effects of the perturbation’s dynamics inside the flow domain. The requirement for the temporal decay of $\Delta A_{rd}$ leads to novel sufficient conditions for the nonlinear stability of the circular Couette flow in response to axisymmetric perturbations. Comparisons with historical studies show that our results shed light on the experimental measurements of Wendt (1933) and significantly extend the classical nonlinear stability results of Serrin (1959) and Joseph & Hung (1971). When the flow is nonlinearly stable and evolves axisymmetrically for all time, then it always decays asymptotically in time to the circular Couette flow determined uniquely by the setup of the rotating cylinders. This study provides new physical insights into a classical flow problem that was studied for decades.

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Date submitted: 19 Jul 2016

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