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Preserving the Helmholtz dispersion relation: One-way acoustic wave propagation using matrix square roots LAURENCE KEEFE, Desondes LLC, West Linn, OR — Parabolized acoustic propagation in transversely inhomogeneous media is described by the operator update equation $U(x, y, z + \Delta z) = e^{ik_0(-1+\sqrt{1+\tilde{Z}}\ )} U(x, y, z)$ for evolution of the envelope of a wavetrain solution to the original Helmholtz equation. Here the operator,$\tilde{Z} = \nabla_T^2 + (n^2 - 1)$, involves the transverse Laplacian and the refractive index distribution. Standard expansion techniques(on the assumption $\tilde{Z} \ll 1$) produce pdes that approximate, to greater or lesser extent, the full dispersion relation of the original Helmholtz equation, except that none of them describe evanescent/damped waves without special modifications to the expansion coefficients. Alternatively, a discretization of both the envelope and the operator converts the operator update equation into a matrix multiply, and existing theorems on matrix functions demonstrate that the complete (discrete) Helmholtz dispersion relation, including evanescent/damped waves, is preserved by this discretization. Propagation-constant/damping-rates contour comparisons for the operator equation and various approximations demonstrate this point, and how poorly the lowest-order, textbook, parabolized equation describes propagation in lined ducts.

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