Cahn-Hilliard Regularization of the "\(\mu(I)\)" Rheology

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\[
\psi(\nabla \mathbf{v}, \nabla \nabla \mathbf{v}) = \psi_0(\mathbf{D}) + k \|
abla \nabla \mathbf{v}\|^2,
\]
with \(\mathbf{D} = \text{Sym}(\nabla \mathbf{v})\) and \(k > 0\), with stress for the standard \(\mu(I)\) model given by \(\partial \psi_0 / \partial \mathbf{D}\), and with hyperstress given by \(\partial \psi / \partial \nabla \nabla \mathbf{v}\). Following the linear-stability analysis of Barker et al. of the momentum balance and continuity equation, we obtain a modification of their dispersion relation giving growth rate in terms of spatial wave number. It is found that the higher-gradient terms in the CH model lead to a large wave number cut-off of the instability, so that the model provides a possibly useful regularization of the \(\mu(I)\) model.