Gravity current into an ambient fluid with an open surface

MARIUS UNGARISH, Technion, Haifa — Consider the steady-state gravity current of height $h$ and density $\rho_1$ that propagates into an ambient motionless fluid of height $H$ and density $\rho_2$ with an upper surface open to the atmosphere (open channel) at high Reynolds number. The current propagates with speed $U$ and causes a depth decrease $\chi$ of the top surface. This is a significant extension of Benjamin’s (1968) seminal solution for the fixed-top channel $\chi = 0$. Here the determination of $\chi$ is a part of the problem. The dimensionless parameters of the problem are $a = h/H$ and $r = \rho_2/\rho_1$. We show that a control-volume analysis determines $\tilde{\chi} = \chi/H$ and $Fr = U/(g'h)^{1/2}$ as functions of $a, r$, where $g' = (r^{-1} - 1)g$ is the reduced gravity. The system satisfies balance of volume and momentum (explicitly), and vorticity (implicitly). We present solutions. The predicted flows are in general dissipative, and thus physically valid only for $a \leq a_{\text{max}}(r) \approx 0.5r$ where non-negative dissipation appears. The open-surface $Fr(a, r)$ is smaller than Benjamin’s $Fr_b(a)$, but the reduction is not dramatic, typically a few percent. In the Boussinesq $r \approx 1$ case, $\tilde{\chi} \ll 1$ while $Fr$ and dissipation are close to Benjamin’s values.